Impact of exposed compartment on modeling disease dynamics

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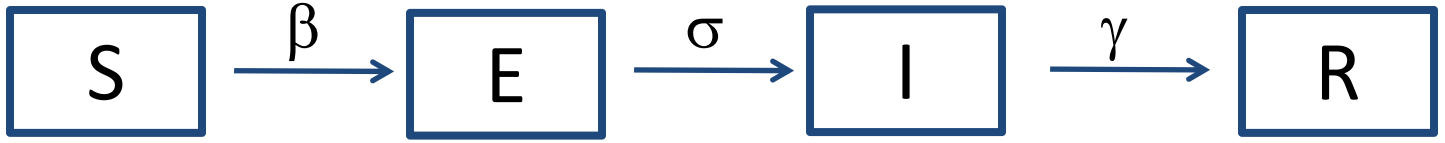
SEIR model

- Use a simplification of model from S. Bianco and K. Hu



- Parameters: $\beta = 0.278/\text{day}$, $\sigma = 0.1/\text{day}$ (mean incubation period 10 days), $\gamma = 0.07/\text{day}$

Fixed vs exponentially distributed exposed time

Exponentially distributed exposed time:

$$\frac{dS}{dt} = -\beta S \frac{I}{N}$$
$$\frac{dE}{dt} = \beta S \frac{I}{N} - \sigma E$$
$$\frac{dI}{dt} = \sigma E - \gamma I$$
$$\frac{dR}{dt} = \gamma I$$

Fixed exposed time:

$$\frac{dS(t)}{dt} = -\beta S(t) \frac{I(t)}{N}$$
$$\frac{dI(t)}{dt} = \beta S(t-\tau) \frac{I(t-\tau)}{N} - \gamma I(t)$$
$$\frac{dR(t)}{dt} = \gamma I(t)$$

Incubation time $\tau = 10$ days

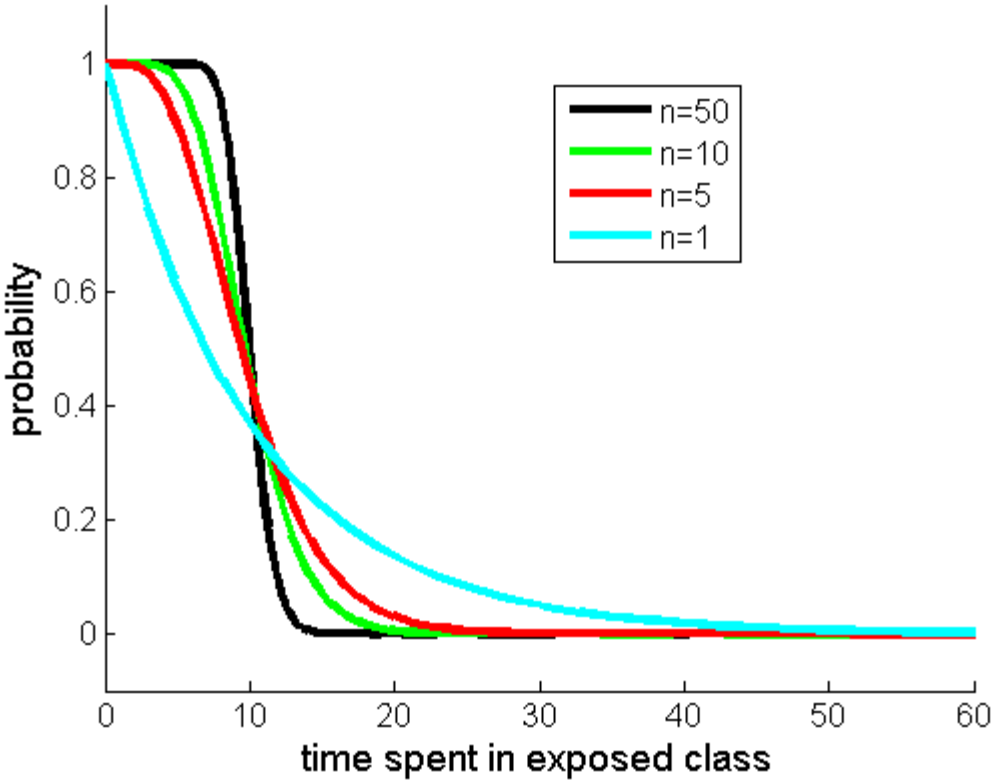
And distributions in between

- With n exposed compartments

$$\frac{dS}{dt} = -\beta S \frac{I}{N}$$
$$\frac{dE_1}{dt} = \beta S \frac{I}{N} - n\sigma E_1$$
$$\ldots$$
$$\frac{dE_i}{dt} = n\sigma E_{i-1} - n\sigma E_i$$
$$\ldots$$
$$\frac{dI}{dt} = n\sigma E_n - \gamma I$$
$$\frac{dR}{dt} = \gamma I$$

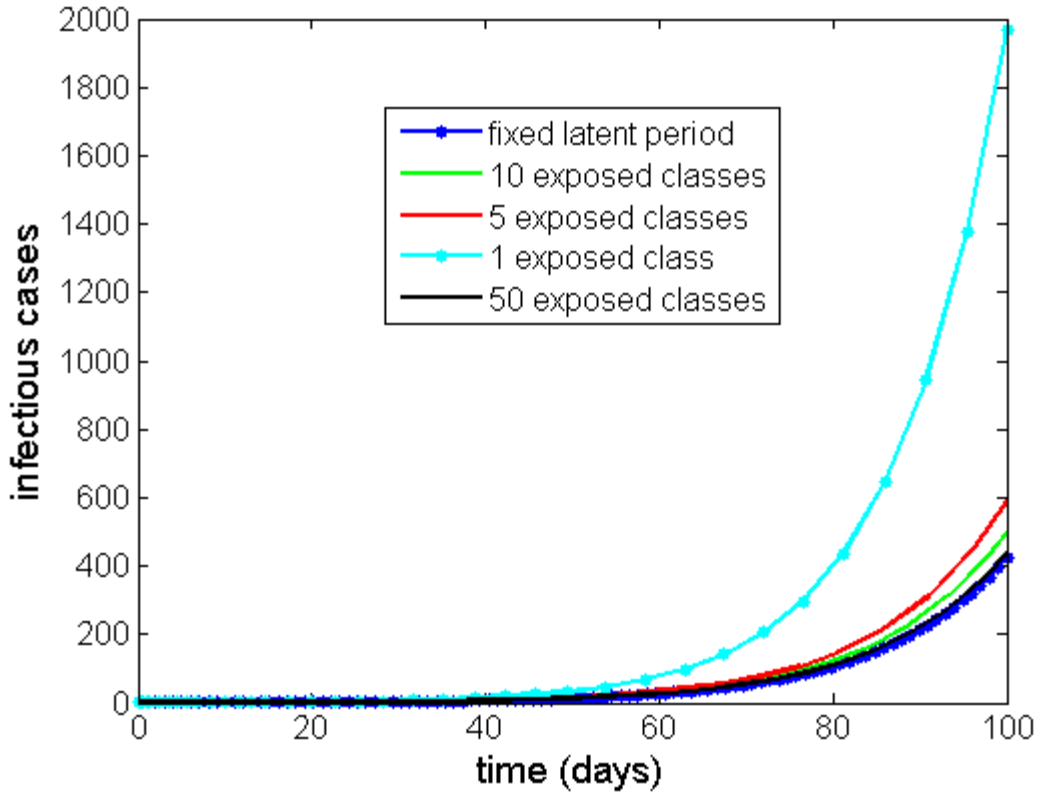
- $n = 1$ is exponentially distributed case
- Approaches fixed exposed time as $n \to \infty$

Probability to remain in exposed class



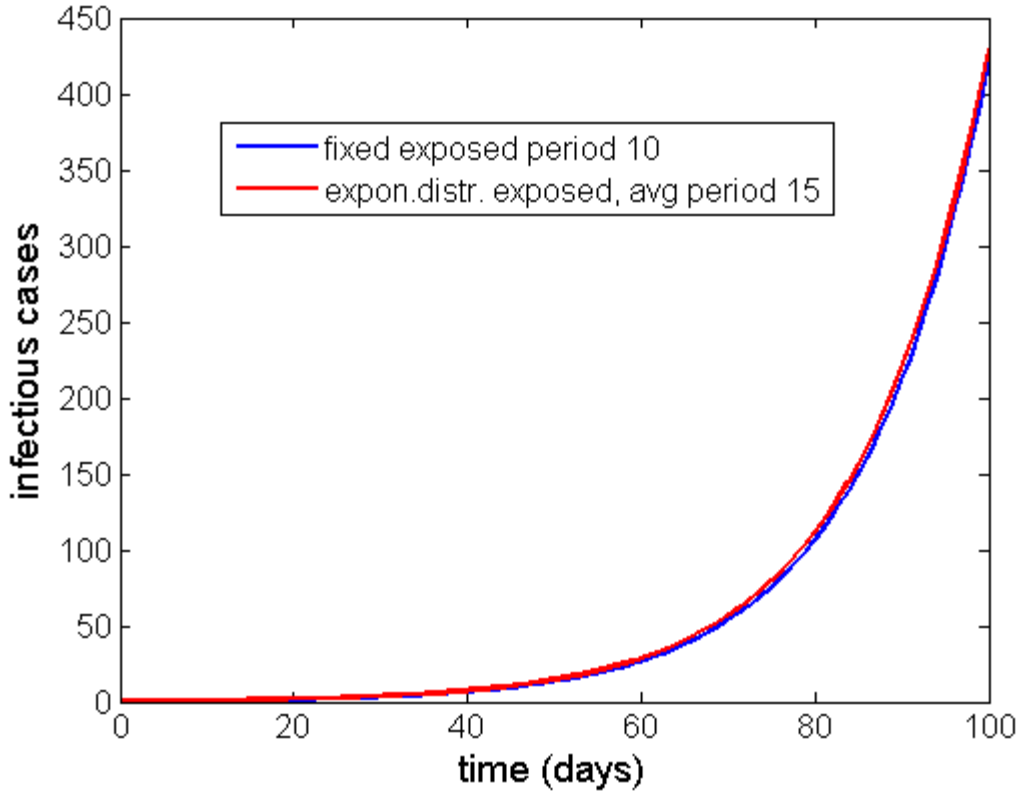
Effect on predicted infection level

- Exponentially distributed exposed period leads to faster infection spread



Effects on parameter fitting

- If in reality there is a fixed exposed period, but fitting is done assuming exponentially distributed exposed times, averaged exposed period can be fit incorrectly
- Here, $\tau = 10$ but mean exposed period of exponential distribution is 15, i.e., $\sigma = 1/15$
- (Fitting done by eye)



Effects on parameter fitting

- "…[M]aking the … assumption of exponentially distributed latent and infectious periods … always results in underestimating the basic reproductive ratio of an infection from outbreak data" – Wearing et al, "Appropriate Models for the Management of Infectious Diseases", PLoS Medicine 2005