LoRaWAN: single gateway capacity for a reasonable traffic

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Capacity of a LoRaWAN cell

- **How many nodes** can a single GW handle?
  - We are looking at **uplink capacity** only!
- **LoRaWAN transmissions**
  - **Aloha** access
    - With **physical capture**!
      - Reception of a given frame if the colliding frame is 6 dB weaker\(^3\)
  - Several spreading factors **SF7** — **SF12**
    - **Quasi-orthogonal** symbols (16 to 36 dB rejection)\(^1\)
    - Transmission duration ~ doubles from **SF**\(_n\) to **SF**\(_{n+1}\)
  - Stringent **duty cycle** limitations (1% in each sub-band)
  - Relatively short frames

  **2.5 s** of time on air at **SF12** for **59 bytes**!

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\(^3\)Dedicated networks for IoT: PHY / MAC state of the art and challenges, C. Goursaud, J.M. Gorce, 2015
SF boundaries

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signals since the system is assumed ergodic (i.e., any two instances of time are statistically independent). Note that the transmit powers of end-devices with the same SF signals are assumed equal. The second outage condition is therefore given by the complement of:

\[ Q_1 = \mathbb{P}\left[ \frac{|h_1|^2 g(d_1)}{|h_k|^2 g(d_k)} \geq 4 \mid d_1 \right], \tag{4} \]

thus providing a statistically meaningful performance metric quantifying when collisions of the same SF are significant. Intuitively, we expect \( Q_1 \) to decay with increasing \( \bar{N} \).

Combined, the two outage conditions form the joint outage probability \( J_1 \) of a received signal \( s_1 \) given by the complement of a successfully received signal defined as \( J_1 = 1 - H_1 Q_1 \).

3) Coverage Probability: The coverage probability is the probability that a randomly selected end-device is in coverage (i.e., not in outage) at any particular instance of time. One may obtain the system’s coverage probability \( \varphi_c \) with respect given by \( f_{d_i}(x) = 2\pi x / |\mathcal{V}(d_i)| \). Calculating the pdf of \( g(d_i) \)

\[ f_{g(d_i)}(x) = \left| \frac{d}{dx} g^{-1}(x) \right| f_{d_i}(g^{-1}(x)) = \frac{\lambda^2 x^{-\frac{n+2}{n}}}{8\eta \pi |\mathcal{V}(d_i)|} \]  

which has a finite support on \( g(l_{j+1}) \leq x \leq g(l_j) \), and recalling that \( |h_i|^2 \sim \exp(1) \), it follows that the pdf of \( X_i \) is

\[ f_{X_i}(z) = \int_{g(l_{j+1})}^{g(l_j)} \frac{f_{g(d_i)}(x)}{x} f_{h_i}(z/x) \, dx = \frac{\lambda^2 z^{-\frac{n+2}{n}}}{8\eta \pi |\mathcal{V}(d_i)|} \left[ \Gamma\left( 1 + \frac{2}{\eta} \frac{z}{g(x)} \right) \right]_{x=l_j}^{x=l_{j+1}}, \tag{9} \]

supported on \( z \in \mathbb{R}^+ \), where \( \Gamma(\cdot, \cdot) \) is the upper incomplete gamma function. Integrating \( (9) \) we arrive at the cdf of \( X_i \)

\[ F_{X_i}(z) = \frac{z^\frac{2}{n} \lambda^2}{16\pi |\mathcal{V}(d_1)|} \left[ \frac{e^{\frac{-z}{g(x)}} - 1}{g(x)^\frac{2}{n}} - \Gamma\left( 1 + \frac{2}{\eta} \frac{z}{g(x)} \right) \right]_{x=l_j}^{x=l_{j+1}} \tag{10} \]
• H1 — Outage due to attenuation
• Q1 (or QI) — Outage due to collision

Several follow-up papers… e.g. taking into account inter-SF interference (Mahmood et al., 2018), antenna diversity (Hoeller et al., 2018)…
The devil is in the details

- **1% duty cycle for all nodes, regardless of SF**
  
  This means that the application changes the amount of data depending of the SF!? — I don’t think so

- Also, there are 3+ channels per band!

- **Collision probability**: given by “the expected number of concurrently transmitting end-devices”: \( N_{\text{nodes}}(SF) \times 1\% \)

  **But this is Aloha!**

  \( \Rightarrow \) the probability of collision is \( 2 \times N_{\text{nodes}}(SF) \times 1\% \)

- H1 on the graph does not match the provided formula
  
  No big deal – the formula does not really make sense anyway

  (a mashup of free space and 2 ray ground)

- **Arbitrary** SF boundaries at 2, 4, 6, 8 and 12 km (really?)
So, can we tidy things up?

• There are at least 3 channels per band ⇒ duty cycle: **0.33%**

• Use **same traffic for all SF:**
  - Saturate SF12
  - 59B, 2.466 s of time on air, 1 packet / 747 s per frequency channel
  - We will be able to repeat this packet **6 times!**
    (3 times in subband **h1.3**, 3 times in **h1.4**)
  - 6 repetitions → **40% PDR** (Packet Delivery Ratio) gives **95% data extraction**
    — with **12 repetitions**, we need only **22% PDR** —

• Okumara Hata propagation model
  (less favorable than anything else)

• Collision probability: use an Aloha/Poisson traffic model **with capture**
  (works just as well as the “theory of order statistics”)
  (Sorry, math nerds…)

• **Which SF** should each node pick? — This is not a detail!

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Aloha with capture in a Rayleigh channel

• Probability of no interference: $\exp(-2v)$
  where $v$: frame arrival intensity $\times$ frame duration

• Probability of single interference, 6dB lower:

  $\frac{2}{5}v \exp(-2v)$

  (The probability that another frame is $x$ times lower is $\frac{1}{x+1}$ with exp. distribution)

• With 2 or more interferers, we consider the frame lost (rare anyway)

• We consider that all nodes in a given SF get similar attenuation
  (mostly wrong for SF7, but there are no collisions, see below)
Regular SF boundaries

- SF boundaries at 2, 4, 6, 8, 10 (and 12) km (more than half of the nodes use SF11 or SF12)
- 500 nodes, Antenna height 15 m, 6 dB gain
Clearly, this channel model does not give a range of 12km!
Let's aim at a range giving empty channel PDR of e.g. **45% for SF12. (9.1 km)**
Let's change of SF as soon as the SNR gives a PDR < 45%
1000 nodes, PDR threshold = 45%
Adjusting SF boundaries 45%/1k nodes

• 5 thresholds to adjust
• Algorithm: Nelder Mead simplex: max(min(PDR(SF)))
8000 nodes, PDR threshold = 90%
Range: 5.3 km
Adjusted boundaries, 90% PDR/8k nodes

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Conclusion

• The smaller the radius, the more nodes can be handled — up to 1000s of nodes!
  → And then, the downlink capacity will be the bottleneck

• The target SNR needs to be more and more discriminating for higher SF

  **Very true for short range and dense cells**

• How do we control the nodes (Network server + ADR MAC messages)?

• Power control in SF7 zone would be much welcome!
  (NB: increasing power then SF is also good in terms of power consumption, see M. N. Ochoa et al., Evaluating LoRa Energy Efficiency for Adaptive Networks: From Star to Mesh Topologies, WiMob 2017)
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  Especially for short range and dense cells
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- What if multiple gateways?