LoRaWAN: single gateway capacity for a reasonable traffic

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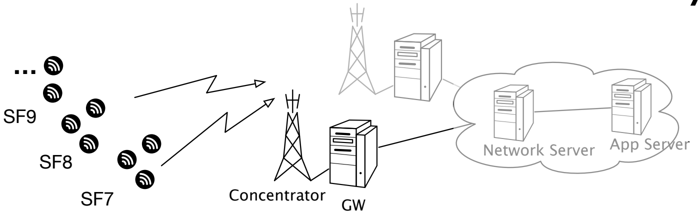
Capacity of a LoRaWAN cell

- **How many nodes** can a single GW handle?
  - ✓ We are looking at **uplink capacity** only!
- LoRaWAN transmissions
  - ✓ **Aloha** access
    - ▸ With **physical capture**!

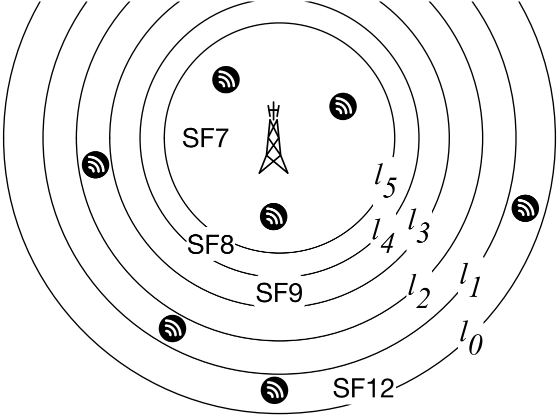
      Reception of a given frame if the colliding frame is 6 dB weaker[3]
  - ✓ Several spreading factors SF7 — SF12
    - ▸ **Quasi-orthogonal** symbols (16 to 36 dB rejection)[1]
    - ▸ Transmission duration $\sim$ doubles from SF_n to SF_{n+1}
  - ✓ Stringent **duty cycle** limitations (**1%** in each sub-band)
  - ✓ *Relatively* short frames

2.5 s of time on air at **SF12** for **59 bytes**!



[3] Dedicated networks for IoT : PHY / MAC state of the art and challenges, C. **Goursaud**, J.M. **Gorce**, **2015**

SF boundaries



Previous work

Low Power Wide Area Network Analysis: Can LoRa Scale?
O. **Georgiou** and U. **Raza**
IEEE Wireless Communications Letters **2017**

signals since the system is assumed ergodic (i.e., any two instances of time are statistically independent). Note that the transmit powers of end-devices with the same SF signals are assumed equal. The second outage condition is therefore given by the complement of:

$$Q_1 = \mathbb{P}\Big[\frac{|h_1|^2 g(d_1)}{|h_{k^*}|^2 g(d_{k^*})} \geq 4 \,\Big|\, d_1\Big], \tag{4}$$

thus providing a statistically meaningful performance metric quantifying when collisions of the same SF are significant. Intuitively, we expect Q_1 to decay with increasing $\bar{N}$.

Combined, the two outage conditions form the joint outage probability J_1 of a received signal s_1 given by the complement of a successfully received signal defined as $J_1 = 1 - H_1 Q_1$.

3) Coverage Probability: The coverage probability is the probability that a randomly selected end-device is in coverage (i.e., not in outage) at any particular instance of time. One may obtain the system's coverage probability $\wp_c$ with respect

given by $f_{d_i}(x) = 2\pi x/|\mathcal{V}(d_1)|$. Calculating the pdf of $g(d_i)$

$$f_{g(d_i)}(x) = \Big|\frac{\mathrm{d}}{\mathrm{d}x} g^{-1}(x)\Big| f_{d_i}\big(g^{-1}(x)\big) = \frac{\lambda^2 x^{-\frac{\eta+2}{\eta}}}{8\eta\pi|\hat{\mathcal{V}}(d_1)|} \tag{8}$$

which has a finite support on $g(l_{j+1}) \leq x \leq g(l_j)$, and recalling that $|h_i|^2 \sim \exp(1)$, it follows that the pdf of X_i is

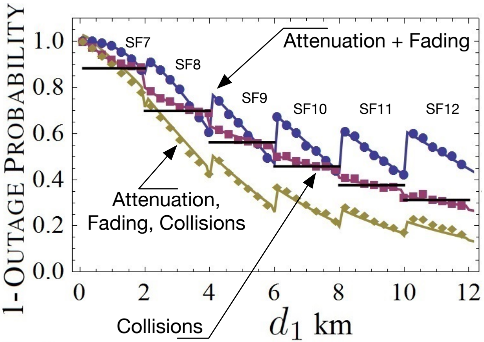
$$\begin{aligned} f_{X_i}(z) &= \int_{g(l_{j+1})}^{g(l_j)} \frac{1}{x} f_{g(d_i)}(x) f_{|h_i|^2}(z/x) \mathrm{d}x \\ &= \frac{\lambda^2 z^{-\frac{\eta+2}{\eta}}}{8\eta\pi|\hat{\mathcal{V}}(d_1)|} \Big[\Gamma\Big(1+\frac{2}{\eta}, \frac{z}{g(x)}\Big)\Big]_{x=l_{j+1}}^{x=l_j}, \end{aligned} \tag{9}$$

supported on $z \in \mathbb{R}^+$, where $\Gamma(\cdot,\cdot)$ is the upper incomplete gamma function. Integrating (9) we arrive at the cdf of X_i

$$F_{X_i}(z) = \frac{z^{\frac{2}{\eta}} \lambda^2}{16\pi|\hat{\mathcal{V}}(d_1)|} \Big[\frac{(e^{\frac{-z}{g(x)}} - 1) z^{\frac{2}{\eta}}}{g(x)^{\frac{2}{\eta}}} - \Gamma\Big(1+\frac{2}{\eta}, \frac{z}{g(x)}\Big)\Big]_{x=l_{j+1}}^{x=l_j} \tag{10}$$

⇑ Check out the cool math formulas! ⇑

Previous work (cont.)



500 nodes

- H1 — Outage due to attenuation
- Q1 (or *Q*1) — Outage due to collision

Several follow-up papers… *e.g.* taking into account **inter-SF interference** (Mahmood *et al.*, 2018), **antenna diversity** (Hoeller *et al.*, 2018)…

The devil is in the details

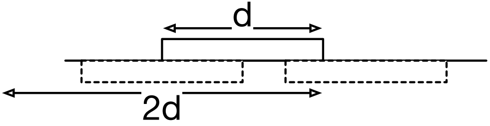
- **1%** duty cycle for **all nodes**, **regardless of SF**

  This means that the application changes the amount of data depending of the SF!? — I don't think so

- Also, there are 3+ channels per band!
- **Collision probability**: given by "the expected number of concurrently transmitting end-devices": $N_{\text{nodes}}(SF) \times 1\%$

  But this is **Aloha**!

  $\Rightarrow$ the probability of collision is $\mathbf{2} \times N_{\text{nodes}}(SF) \times 1\%$



- H1 on the graph does not match the provided formula

  No big deal – the formula does not really make sense anyway

  (a mashup of free space and 2 ray ground)

- **Arbitrary** SF boundaries at 2, 4, 6, 8 and 12 km (really?)

So, can we tidy things up?

- There are at least 3 channels per band $\Rightarrow$ duty cycle: **0.33%**
- Use **same traffic for all SF**:
  - ✓ Saturate SF12
  - ✓ 59B, 2.466 s of time on air, 1 packet / 747 s per frequency channel
  - ✓ We will be able to repeat this packet **6 times**!

    (3 times in subband **h1.3**, 3 times in **h1.4**)
  - ✓ 6 repetitions $\rightarrow$ **40%** PDR (Packet Delivery Ratio) gives **95% data extraction**

    — with **12 repetitions**, we need only **22% PDR** —
- Okumara Hata propagation model

  (less favorable than anything else)
- Collision probability: use an Aloha/Poisson traffic model **with capture**

  (works just as well as the "theory of order statistics")

  (Sorry, math nerds…)
- **Which SF** should each node pick? — This is not a detail!

Aloha with capture in a Rayleigh channel

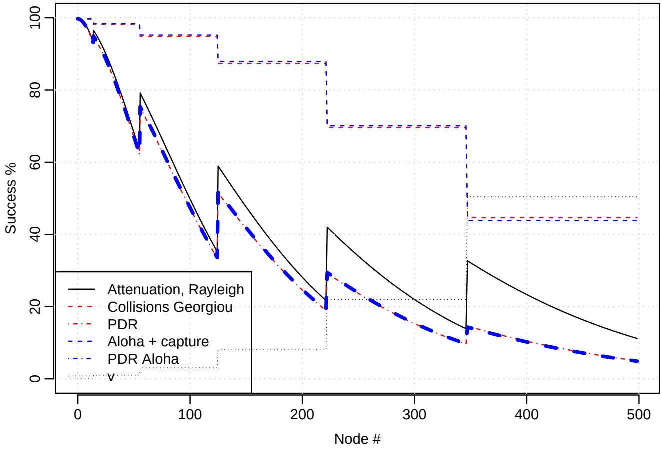
- Probability of no interference : $\exp(-2v)$
  where v: frame arrival intensity $\times$ frame duration
- Probability of single interference, 6dB lower:

$$\frac{2}{5} v \exp(-2v)$$

  (The probability that another frame is x times lower is $\frac{1}{x+1}$ with exp. distribution)
- With 2 or more interferers, we consider the frame lost (rare anyway)
- We consider that all nodes in a given SF get similar attenuation (mostly wrong for SF7, but there are no collisions, see below)

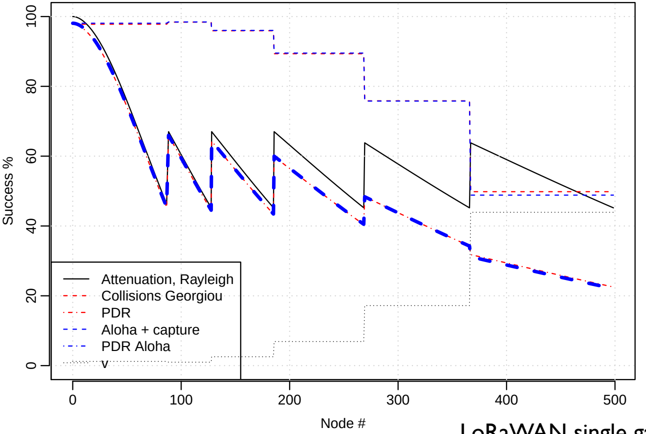
Regular SF boundaries

- SF boundaries at 2, 4, 6, 8 , 10 (and 12) km (more than half of the nodes use SF11 or SF12)
- 500 nodes, Antenna height 15 m, 6 dB gain



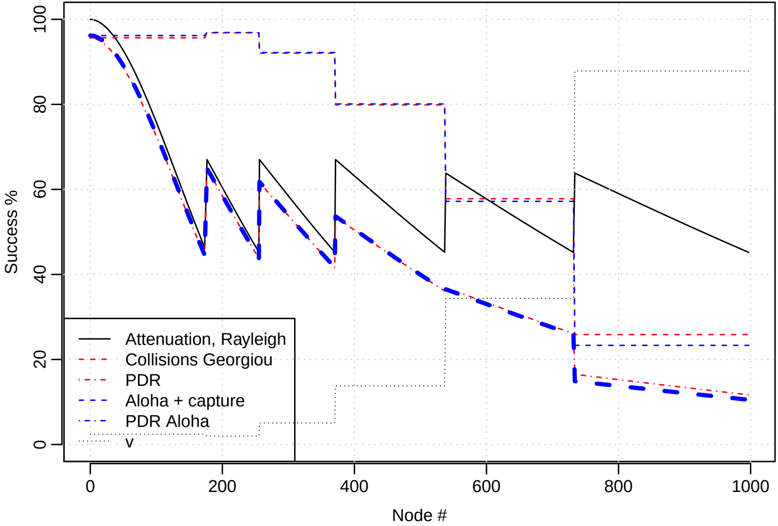
Hum —

- Clearly, this channel model does not give a range of 12km!
- Let's aim at a range giving empty channel PDR of e.g. **45% for SF12**. (**9.1 km**)
- Let's change of SF as soon as the SNR gives a PDR < 45%



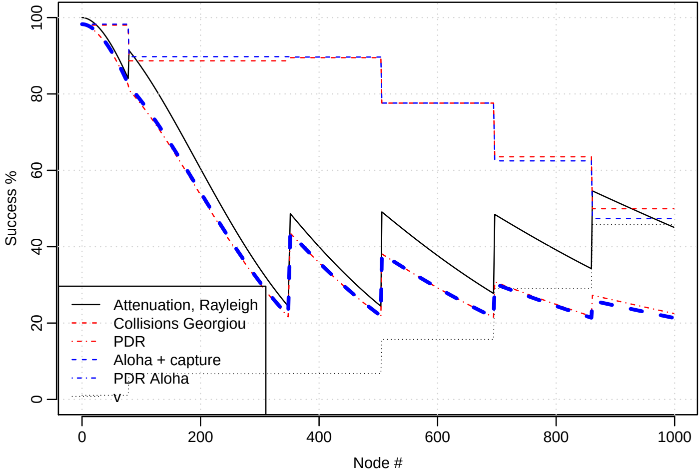
500nodes
3.8 km
4.6 km
5.6 km
6.7 km
7.8 km
9.1 km

1000 nodes, PDR threshold = 45%

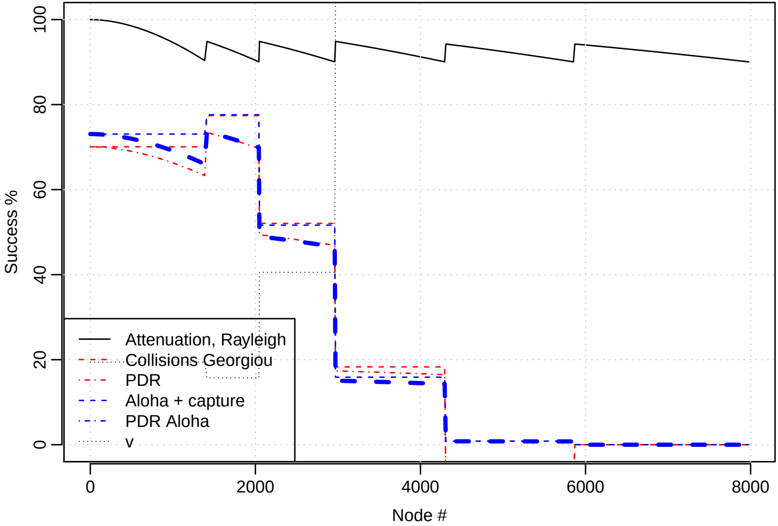


Adjusting SF boundaries 45%/1k nodes

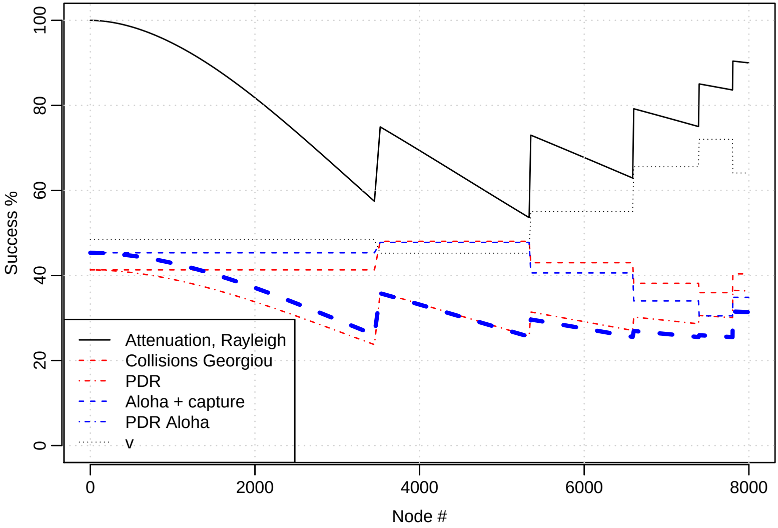
- 5 thresholds to adjust
- Algorithm: Nelder Mead simplex: max(min(PDR(SF))



8000 nodes, PDR threshold = 90%
Range: 5.3 km



Adjusted boundaries, 90% PDR/8k nodes



Conclusion

- The smaller the radius, the more nodes can be handled
  — up to 1000s of nodes!
  $\rightarrow$ And then, the downlink capacity will be the bottleneck
- The target SNR needs to be more and more discriminating for higher SF
  Very true for short range and dense cells
- How do we control the nodes (Network server + ADR MAC messages)?
- Power control in SF7 zone would be much welcome!
  (NB: increasing power then SF is also good in terms of power consumption, see M. N. Ochoa *et al.*, Evaluating LoRa Energy Efficiency for Adaptive Networks: From Star to Mesh Topologies, WiMob 2017)

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- What if multiple gateways?